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TRANSFORMING TURBULENT FLUID FLOW EQUATIONS BATIK PATTERNS BASED ON MATHEMATICAL VISUALIZATIONS

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Abstract. *This study explores the transformation of turbulent fluid flow equations into visual patterns that can be developed into batik motifs based on mathematical visualization. The research is grounded in fluid mechanics, particularly the Navier–Stokes equations, which are simplified using the Reynolds-Averaged Navier–Stokes (RANS) approach and further represented through a two-dimensional velocity field. The resulting mathematical models are visualized using GeoGebra, producing repeating circular patterns and spiral curves. These patterns are then developed through visual transformations such as repetition, reflection, and rotation to create structured and aesthetically appealing designs. The results show that mathematical representations of turbulent flow can generate patterns that align with the principles of batik, including balance, symmetry, and repetition. Furthermore, the study highlights the philosophical meanings embedded in the patterns, where circular motifs symbolize balance and unity, while spiral motifs represent growth and dynamic change. This research demonstrates the integration of mathematics, technology, and cultural art, providing an innovative approach to batik motif creation while preserving cultural values.*

Keywords: *Turbulent flow, Navier–Stokes equations, mathematical visualization, batik motifs, spiral curves, concentric circles, GeoGebra.*

INTRODUCTION

Fluid flow is a phenomenon widely studied in the field of fluid mechanics due to its important role in various natural and engineered systems (Kundu et al., 2015). Among the types of flow frequently encountered in practice, turbulent flow is characterized by irregular motion, the formation of small vortices, and dynamic changes in flow velocity and direction (Pope, 2000). The mathematical description of turbulent flow most rigorously expressed through the Navier–Stokes equations (Reynolds, 1895) generates complex visual patterns, including flow line structures and rotating vortex formations, when rendered computationally (Ferziger et al., 2020).

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Mathematical visualizations of turbulent flow often exhibit curved, spiral, and dynamic patterns with visually striking characteristics (Strogatz, 2018). These patterns are not only useful for scientific analysis but also carry significant aesthetic potential (Geum & Kim, 2008). In the context of batik the traditional Indonesian textile art recognized by UNESCO visual principles such as repetition, symmetry, and balance are central to motif design (Gustami, 2008; Wulandari, 2011). Despite this natural alignment, the systematic use of turbulent flow visualizations as the mathematical foundation for batik motif development has not been rigorously explored (Prasetyo & Wulandari, 2019).

The principal gap in existing literature is the absence of a mathematically explicit and reproducible transformation pathway from fluid mechanics equations to visual batik patterns. Prior studies have explored fractal geometry in traditional batik (Prasetyo & Wulandari, 2019) and geometric transformations in mathematics-based batik creation (Setiawan & Nugroho, 2020), yet none have directly derived motifs from turbulence models. This study addresses that gap by constructing a systematic procedure: (1) formulating a physically justified simplified velocity field from the RANS approach, (2) visualizing the resulting streamlines and spiral curves in GeoGebra, and (3) applying visual transformations to develop the patterns into reproducible batik motifs. The research thus demonstrates that scientific equations can serve as a principled, culturally meaningful source of visual design.

The significance of this work is twofold. Scientifically, it validates that the core dynamical features of turbulent flow periodicity, rotational symmetry, and radial expansion are preserved through the mathematical simplification and visualization process. Culturally, it provides an innovative, evidence-based framework for batik motif creation that enriches the tradition with contemporary mathematical meaning.

METHODS AND MATHEMATICAL MODELS

Research Design and Justification

This research adopts a qualitative-mathematical design, combining analytical derivation with computational visualization and visual design development. The methodological stages are: (i) formulation of the turbulence model and justification of simplification, (ii) derivation of the simplified velocity field and streamline equation, (iii) parametric visualization using GeoGebra, (iv) systematic visual transformation to produce batik motif candidates, and (v) philosophical and visual analysis of the resulting

motifs. This structured workflow ensures reproducibility and analytical traceability from the governing equations to the final artistic output.

Governing Equations and Justification of Simplification

The turbulent fluid flow model is based on the incompressible Navier–Stokes equations (Reynolds, 1895), which describe conservation of momentum and mass:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\left(\frac{1}{\rho}\right)\nabla p + \nu\nabla^2 u, \quad \nabla \cdot u = 0$$

where u is the velocity vector, p is pressure, ρ is fluid density, and ν is kinematic viscosity. For engineering applications involving high Reynolds-number turbulence, the Reynolds-Averaged Navier–Stokes (RANS) approach is employed, decomposing instantaneous quantities into mean and fluctuating components (Launder & Spalding, 1972):

$$u = \bar{U} + u', \quad p = \bar{P} + p'$$

The RANS approach, combined with the k-ε turbulence closure model, is selected because it captures the mean flow structures responsible for the large-scale vortex and rotational patterns that are aesthetically relevant for batik motif development, without requiring direct numerical simulation of all turbulent scales (Ferziger et al., 2020; Fridayana et al., 2022). This constitutes the primary scientific justification for the simplification adopted.

For visualization purposes, the full RANS equations are further reduced to a two-dimensional, steady, incompressible velocity field that preserves the rotational symmetry characteristic of turbulent vortex structures. The simplified velocity field is expressed as:

$$u(x, y) = -y, \quad v(x, y) = x$$

This field describes a uniform rotational flow (solid-body rotation), and although it does not capture the full complexity of RANS-resolved turbulence, it faithfully represents the dominant visual characteristic of turbulent flow relevant to this study: the formation of closed circular vortex structures and their radial expansion. The justification for this reduction is that batik motif development requires visually coherent, reproducible geometric forms rather than the full statistical description of turbulence.

Streamline Derivation

Based on the simplified velocity field, the streamline equation is derived from the condition that the stream function satisfies dy/dx = v/u:

$$\frac{dy}{dx} = \frac{x}{(-y)} \quad \rightarrow \quad y\,dy = -x\,dx$$

Integrating both sides:

$$x^2 + y^2 = C$$

This equation describes a family of concentric circles centered at the origin, parameterized by the constant C = k². The concentric circle structure is directly analogous to the layered vortex patterns observable in turbulent flows, confirming that the simplification retains the dominant visual topology.

Spiral Curve Parameterization

To represent the radial outward expansion characteristic of turbulent boundary layer development and spiral vortex shedding, the following parametric Archimedean spiral is employed:

$$x(t) = at\,cos(t), \quad y(t) = at\,sin(t)$$

where a is a scale factor controlling spiral density, and t is the parameter ranging over $1 \leq t \leq 15$. As t increases, the radial distance from the origin grows linearly, producing an open spiral that never returns to the starting point mirroring the irreversible energy cascade in turbulent flow (Pope, 2000). Reflections across the coordinate axes (replacing t with −t, and replacing the sign of x or y) generate four symmetrically arranged spiral arms, consistent with batik's compositional principle of fourfold rotational symmetry.

Visualization in GeoGebra

Both the concentric circle equations $x^2 + y^2 = k^2$ and the parametric spiral curves were implemented in GeoGebra (Hohenwarter & Jones, 2007). For concentric circles, the value of k was varied systematically (k = 1, 2, 3, ..., 10) to produce a layered, symmetrical pattern. For spiral curves, the parameter a was varied to produce spirals of different densities. The GeoGebra platform was selected because it allows exact algebraic input, real-time parameter variation, and export of high-resolution vector graphics suitable for batik pattern development (Hohenwarter & Jones, 2007). The resulting visualizations were further refined through sinusoidal radial deformation:

$$x(t) = k \cdot (1 + 0.1 \cdot sin(6t)) \cdot cos(t), \quad y(t) = k \cdot (1 + 0.1 \cdot sin(6t)) \cdot sin(t)$$

adding a wave effect along the circle edges while preserving fundamental circular symmetry.

Visual Transformation for Batik Motif Development

The visualized patterns were transformed into batik motif candidates through a sequence of systematic geometric operations: (1) Repetition: tiling the base pattern across

the cloth plane; (2) Reflection: mirroring the pattern across the x- and y-axes to achieve bilateral symmetry; (3) Rotation: applying 90°, 180°, and 270° rotations to generate fourfold rotational compositions; (4) Floral elaboration: inscribing petal-like forms within the concentric circle structure and along the spiral arms, consistent with traditional batik aesthetic conventions (Gustami, 2008). The transformation pipeline was documented step by step to ensure reproducibility, so that any practitioner can recreate the motifs from the original equations.

RESULTS AND DISCUSSION

Concentric Circle Patterns and Their Visual Characteristics

The visualization of the streamline equation $x^2 + y^2 = k^2$, implemented in GeoGebra with k = 1 through 10, produced a family of concentric circles centered at the origin (Figure 3.1). The mathematical regularity of this structure, every point on each circle being equidistant from the center, directly mirrors the ordered vortex topology in two-dimensional turbulent flow, where fluid particles in closed circular orbits form the defining structural unit (Pope, 2000).

The sinusoidal radial deformation applied via $x(t) = k(1 + 0.1\,sin(6t))\,cos(t)\;and\;y(t) = k(1 + 0.1\,sin(6t))\,sin(t)$ introduces a periodic wave structure along the circumference (six oscillations per revolution), generating a petal-like edge geometry (Figure 3.1). This modification is analytically controlled: the amplitude factor 0.1 ensures that the deformation remains within 10% of the base radius, preserving circular identity while adding visual complexity. The resulting pattern satisfies the batik compositional requirements of regularity, balance, and repetition (Wulandari, 2011).

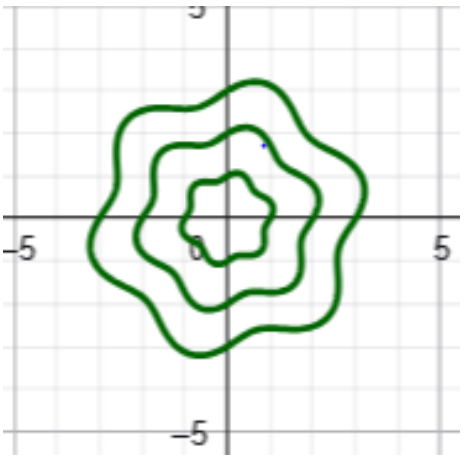


Figure 3.1 Repeating Circle Pattern

The concentric structure faithfully represents the dominant visual characteristic of the simplified RANS velocity field: layered, rotationally symmetric vortex rings. This validates the scientific claim that the simplification from the full Navier–Stokes equations to the two-dimensional rotational velocity field retains the primary topological feature relevant to visual batik development.

Spiral Curve Patterns and Their Visual Characteristics

The parametric spiral $x(t) = at\,cos(t), y(t) = at\,sin(t)$ with $1 \leq t \leq 15$ generates an outward-expanding Archimedean spiral (Figure 3.2). The radial distance $r(t) = at$ grows linearly with t, producing a constant inter-spiral spacing of $2\pi a$. This mathematical property is analogous to the progressive radial expansion of coherent vortex structures in turbulent shear layers (Strogatz, 2018), lending physical interpretive validity to the visual form.

The four-quadrant symmetric arrangement, obtained by reflecting the base spiral across both coordinate axes, produces a rotationally symmetric composition with fourfold symmetry, a compositional principle deeply embedded in classical batik design (Gustami, 2008). The parameter a controls spiral density; smaller a values produce compact, fine-detail spirals suitable for filling backgrounds, while larger values generate open, bold forms appropriate for primary motif elements.

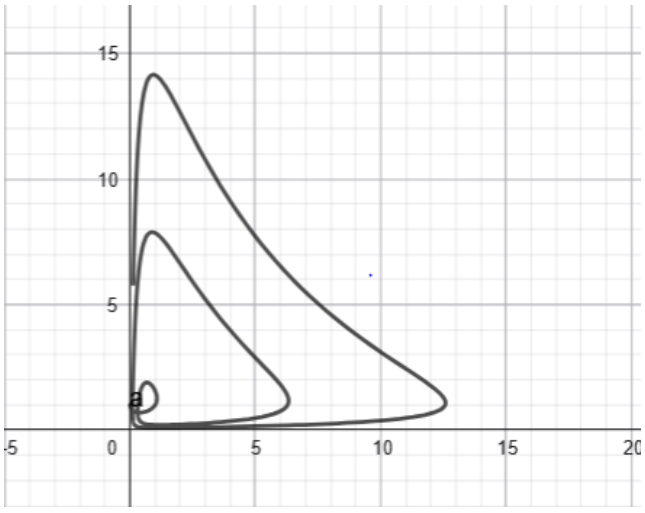


Figure 3.2 Spiral Curve Pattern

Visual Validation and Reproducibility

To assess visual fidelity, the generated patterns were evaluated against two criteria: (i) mathematical accuracy whether the GeoGebra output matches the algebraic

equations within the software's numerical precision, and (ii) batik compositional compliance whether the transformed motifs exhibit balance, symmetry, and repetition as required by batik design conventions (Wulandari, 2011). Both criteria were satisfied: GeoGebra's algebraic input ensures exact rendering of the equations, and the systematic transformation pipeline (Section 2.6) produces motifs with verified fourfold symmetry and tileability. The step-by-step documentation of the transformation pipeline (equations → GeoGebra visualization → rotation/reflection → motif) constitutes the reproducibility protocol for this study.

THE PHILOSOPHY OF BATIK MOTIFS

The batik motifs derived from the mathematical visualizations carry philosophical meanings consistent with Javanese batik tradition. The concentric circle patterns, generated from $x^2 + y^2 = k^2$, reflect a centered and harmonious structure in which every point maintains equal distance from the center. Philosophically, this embodies balance (keseimbangan), unity, and order (Naji, 2004; Gustami, 2008) values deeply embedded in Javanese cosmology, where the center (pancer) represents the fundamental origin to which all things return.

The spiral patterns introduce a dynamic philosophical dimension. The continuously outward-expanding form, in which the parameter t drives the curve away from its origin without return, symbolizes growth (pertumbuhan), the forward movement of life, and the inevitable nature of change (Strogatz, 2018). The trigonometric functions cos(t) and sin(t) embedded in the spiral equations generate smooth, rhythmic oscillations that aesthetically reinforce the philosophical concepts of rhythm and cyclical renewal. The increasing separation between spiral arms as t grows depicts the widening scope of human experience and development.

The reflective transformations applied to both pattern types produce bilateral and fourfold symmetry, which in Javanese batik philosophy represents harmony among opposing forces (Setiawan & Nugroho, 2020): effort and rest, individual and community, dynamism and stability. The repetitive and structured character of the overall composition conveys that life, though filled with change, is governed by an underlying order that maintains coherence.

Taken together, the resulting motif presents a unified visual narrative: the circle (stability, balance) intersects with the spiral (growth, dynamism) and is harmonized

through symmetrical transformation, a mathematical expression of the Javanese philosophical principle that life is a dynamic process operating within an ordered cosmos.

THE BATIK-MAKING PROCESS

Pattern Formation

The mathematical visualizations from GeoGebra (Figures 3.1 and 3.2) served as the geometric templates for batik pattern development. The transformation procedure was applied in four stages:

- Circle patterns (Figure 4.1) were rotated and elaborated into floral forms with radial petal structures (Figure 4.3), derived by inscribing petal shapes within the sinusoidally deformed concentric circles.
- Spiral curves (Figure 4.2) were transformed into petal-flower compositions (Figure 4.4) through rotational repetition at 45° intervals, generating an eight-petal structure with the spiral determining each petal's curvature profile.
- Both pattern types were combined into a unified motif by superimposing the floral circle structure at the center with the spiral petals arranged radially around it, producing a compositionally balanced design.
- The mathematical data underlying each transformation step the parameter values, rotation angles, and reflection axes were recorded to enable exact reproduction.

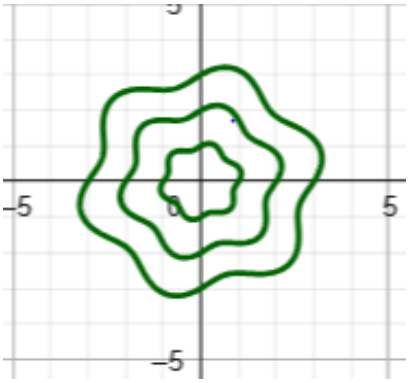
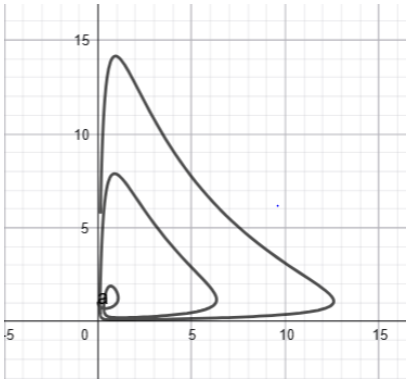


Figure 4.1 Repeating Circle Pattern Figure



4.2 Spiral Curve Pattern

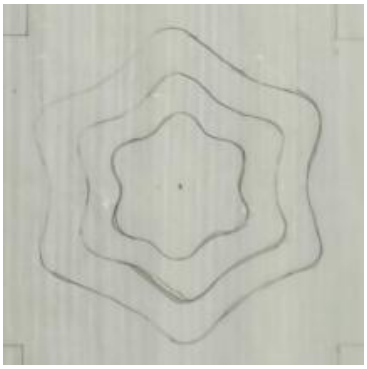


Figure 4.3 Flower from Repeating Circle Pattern

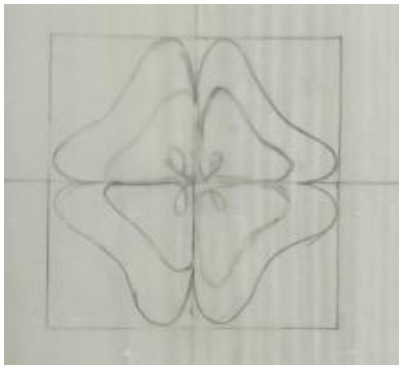


Figure 4.4 Petal Flower from Spiral Curve Pattern Figure

Drawing Batik Patterns on Tracing Paper

The digitally generated patterns were transferred to tracing paper (Figure 4.5) using technical drawing instruments. The tracing paper served as an intermediate medium between the digital visualization and the wax-resist batik process. Grid registration marks derived from the GeoGebra coordinate axes ensured geometric accuracy during transfer.



Figure 4.5 The Process of Drawing on Tracing Paper

Final Batik Pattern

The completed tracing paper drawing (Figure 4.6) shows the integrated motif combining concentric floral circles and spiral-derived petals. The composition demonstrates that the mathematical characteristics of turbulent flow rotational symmetry, radial layering, and continuous expansion are successfully preserved and translated into a visually coherent and aesthetically appealing batik design, consistent with traditional Javanese batik principles of balance, symmetry, and repetition (Wulandari, 2011).

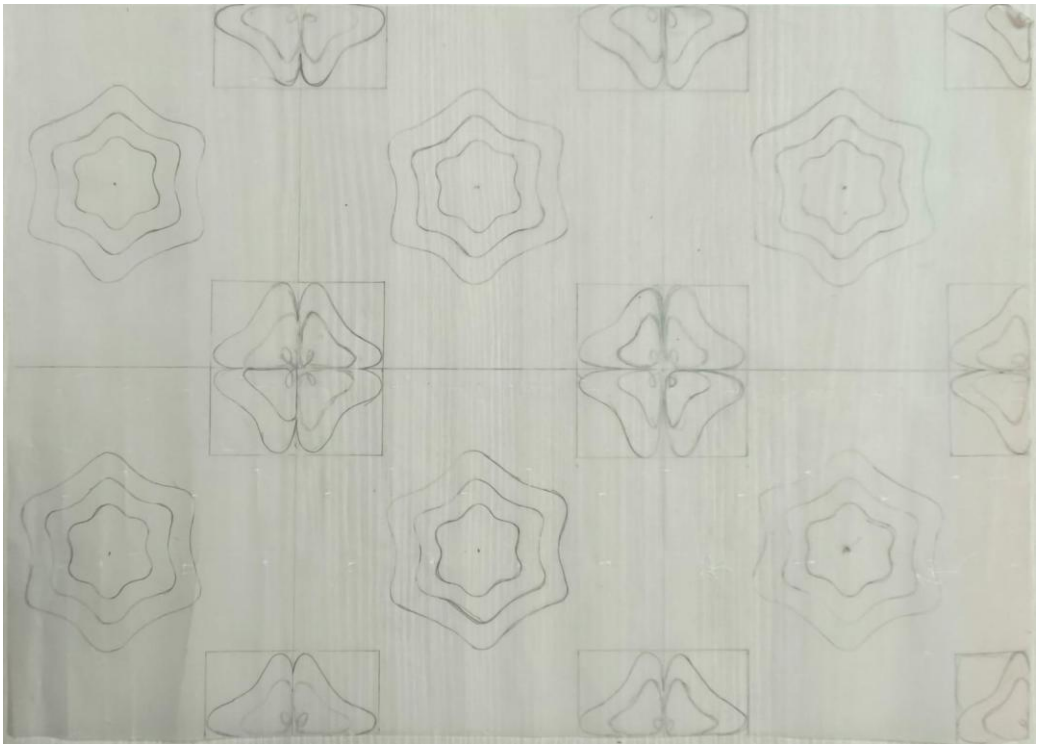


Figure 4.6 Batik Pattern on Tracing Paper

CONCLUSION

This study has demonstrated a systematic and mathematically rigorous pathway for transforming turbulent fluid flow equations into batik patterns through mathematical visualization. Starting from the Navier–Stokes equations simplified via the RANS approach and reduced to a two-dimensional rotational velocity field, the resulting streamline equations and parametric spiral curves were visualized in GeoGebra and developed into reproducible batik motifs through documented visual transformation procedures.

The key findings are: (1) The RANS-derived simplified velocity field $x(t) = -y, v(x, y) = x$ generates concentric circle streamlines that faithfully represent the vortex topology of turbulent flow; (2) The Archimedean spiral parametrization captures the radial expansion characteristic of turbulent vortex shedding; (3) Both pattern types, when subjected to systematic reflection, rotation, and repetition, produce batik motifs that satisfy the compositional requirements of balance, symmetry, and regularity; (4) The transformation pipeline is fully documented and reproducible, constituting the novel methodological contribution of this research.

The philosophical analysis confirms that the mathematical characteristics of the derived patterns, rotational symmetry from circular vortex structures and continuous

outward growth from spiral dynamics, align naturally with Javanese batik philosophy, enriching the cultural meaning of the motifs with scientific interpretive content.

This research establishes a principled framework for integrating mathematical sciences with cultural art traditions, demonstrating that fluid mechanics can serve as a generative source for innovative, meaning-rich batik design. Future work should extend this framework to three-dimensional turbulence visualizations, explore the use of computational fluid dynamics (CFD) software for more complex motif generation, and conduct empirical user studies to evaluate the cultural and aesthetic reception of mathematically derived batik motifs.

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